The Non-User's Pocket Guide to the Transient Knowledge Necessary for the Structural Divisions of the Architect Registration Exam- ARE

CONCEPT	FORMULAE AND DIAGRAMS	COMMENTS

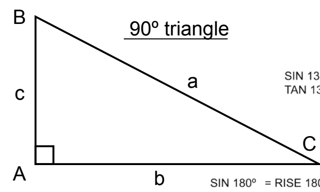
Trigonometry/Math

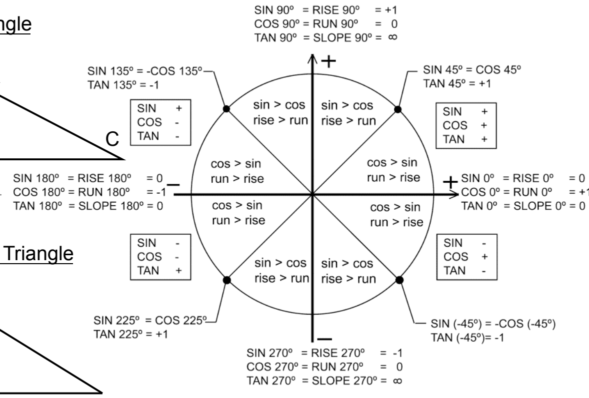
Memory Trick: SOHCAHTOA (Indian Tribe)

$\text{Sin } C = \dfrac{OPP}{HYP}$ or $\dfrac{Rise}{Slope}$ or $\dfrac{c}{a}$ (Rise)

$\text{Cos } C = \dfrac{ADJ}{HYP}$ or $\dfrac{Run}{Slope}$ or $\dfrac{b}{a}$ (Run)

$\text{Tan } C = \dfrac{OPP}{ADJ}$ or $\dfrac{Run}{Rise}$ or $\dfrac{c}{b}$ (Slope)





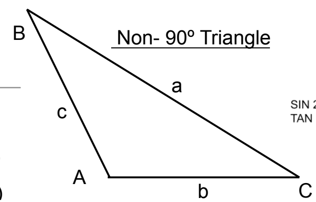
- used when triangle has a 90º angle.
- SIN ⟹ RISE
- COS ⟹ RUN
- TAN ⟹ SLOPE
- SIN and COS of any angle are between (+/-) 1
- 0º < angle < 45º ⟹ COS > SIN
- 45º < angle < 90º ⟹ SIN > COS

Law of Sines

$$\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$$

Law of Cosines

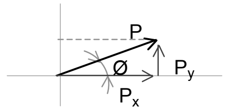
$$a^2 = b^2 + c^2 - 2bc\,(\text{Cos } A)$$
$$b^2 = a^2 + c^2 - 2ac\,(\text{Cos } B)$$
$$c^2 = a^2 + b^2 - 2ab\,(\text{Cos } C)$$



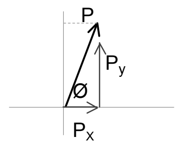
- Law of Sines and Cosines are used when triangle has no right angles.
- Law of Sines is used when you are given more angles than sides.
- Law of Cosines is used when you are given more sides than angles

Forces

Variations in L.O.A.

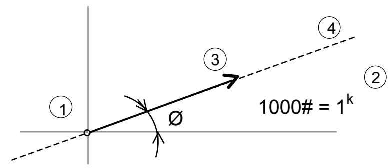


- Shallower angles (<45º) have larger horizontal components

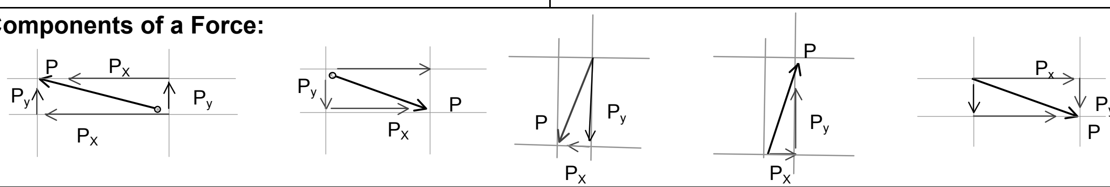


- Steeper angles (>45º) have larger vertical components

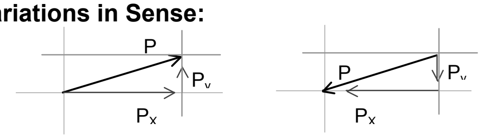
Properties of a Force:



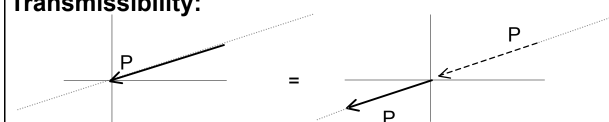
Components of a Force:



Variations in Sense:



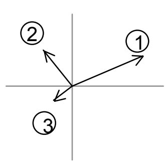
Transmissibility:



Force Addition:

Algebraic Method:

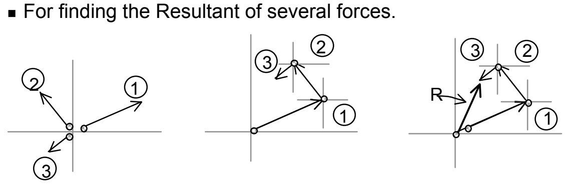
- For finding the resultant of several forces



Force	Horizontal	Vertical
1	+/-	+/-
2	+/-	+/-
3	+/-	+/-
R	+/- $R_X = \Sigma X$	+/- $R_y = \Sigma Y$

Graphic Method for Force Addition:

- For finding the Resultant of several forces.



- Tails at same P.O.A.
- Tail of 2 on Head of 1
- Tail of 3 on Head of 2
- Resultant begins at 1[s] Tail and ends at last Head

Comments:

- A Force is defined by four properties:
  1. Point of Application (P.O.A.)
  2. Magnitude (#,kips)
  3. Sense (Arrowhead, Push or Pull, C or T)
  4. Line of Action (L.O.A.) , (Angle with horizontal)
- The Resultant of several forces is a single force that has the same effect on a body as all the other forces combined.
- The Resultant is also a force and is thus defined by the four properties listed above.
- The Equilibrant is also defined as a force that has the same P.O.A., Magnitude and L.O.A. as the Resultant but has an opposite sense (Arrow)
- Algebraic Method for finding the Resultant of several forces is used when force magnitudes and lines of action for each force are known
- Algebraic Method of Force Addition
  1. Resolve each force into vertical and horizontal components
  2. The algebraic (+/-) sum of all horizontal components gives the horizontal component of the Resultant.
  3. The algebraic (+/-) sum of all vertical components gives the vertical component of the Resultant
- Graphic Method is used when a system is in equilibrium and we need to calculate one or more unknown forces that contribute to equlibrium
- Graphic Method for Force Addition
  1. Arrange all forces Head to Tail then add (independent of order)
  2. Resultant begins with its Tail at the Tail of the 1[st] Force and Head at the Head of the last
  3. Resultant can be determined through calculation

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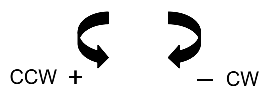
CONCEPT	FORMULAE AND DIAGRAMS	COMMENTS

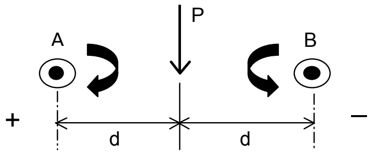
Moments and Couples

Moment

Moment = Force X ⊥ Distance



Force P creates a Positive Moment about point A



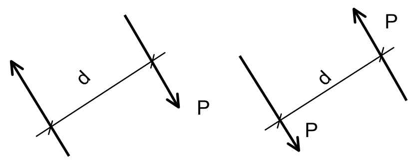
Force P creates a Negative Moment about point B

- A & B are called Centers of Moment, or Centers of Rotation
- The perpendicular distance (d) is called the Moment Arm, or Lever
- Summing Moments ($\sum M = 0$) to establish equilibrium
- To find Beam / Truss reactions
- To maintain equilbrium of members
- Overturning Moments due to Wind Loads or Hydrostatic Pressure

Couple

Moment of a Couple= P x d

(clockwise, CW)

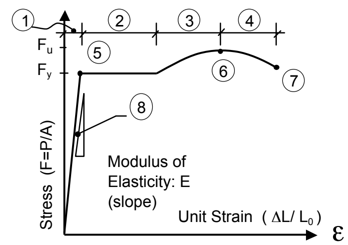


(CCW)

- Unlike a Moment, a Couple is NOT about a certain point, but rather it is about ANY and ALL points.
- A Couple depends on Force (P), and perpendicular distance (d) between two Forces that make up the couple.
- Couple between top Chord (C) and bottom chord (T) in a simply supported truss
- Couple between compression in concrete (top) and tension in rebar (bottom) of reinforced beam

Stress / Strain

Formulas			Units
F: $\frac{P}{A}$	$\Rightarrow$	Direct Stress	PSI
ε: $\frac{\Delta L}{L_o}$	$\Rightarrow$	Unit Strain	in / in
E: $\frac{F}{\varepsilon}$	$\Rightarrow$	Modulus of Elasticity= Stress / Strain	PSI



1. ELASTIC RANGE: straight line relationship, slope = E
2. PLASTIC RANGE: increase in strain, no increase in Load / Stress
3. STRAIN HARDENING: material deforming in section (necking), and in length
4. FAILURE: Material is gone!
5. YIELD POINT/ YIELD STRENGTH: material is no longer elastic, deformation is permanent
6. ULTIMATE STRENGTH: material is about to fail
7. RUPTURE: Kiss it Good-Bye
8. E: Modulus of Elasticity.Measures material's resistance to deformation

Axial Loads

$$\Delta L = \frac{PL_0}{AE}$$

ΔL: deformation, changes in Length (in) caused by Axial Load (P)

P : Axial Load (#,k)

L_0 : Original, undeformed Length (in. not ft.)

A : Cross Sectional Area (in^2)

E : Modulus of Elasticity (PSI, KSI)

- E_{A36},A_{-50}= 29,000 KSI
- ↗ P $\Rightarrow$ ↗ ΔL
- ↗ L_0 $\Rightarrow$ ↗ ΔL
- ↗ A $\Rightarrow$ ↘ ΔL
- ↗ E $\Rightarrow$ ↘ ΔL

$\Delta L = \alpha\ (\Delta T)\ L_0$

ΔL: Deformation, change in length (in), caused by change in temperature (ºF)

ΔT: Change in temperature

α : Coefficient of thermal expansion/contraction

- Shortening or Elongation of members along their axis
- Change (Expansion & Contraction) of shape due to Temperature
- Examples include Columns, Trusses, Cables, Cross Bracing

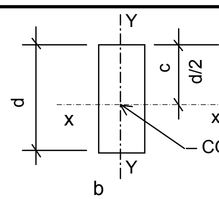
Geometry

A = bd	$\Rightarrow$	Area (In^2)	$\Rightarrow$	Shear
$I_{xx} = \frac{bd^3}{12}$	$\Rightarrow$	Moment of Inertia (In^4)	$\Rightarrow$	Deflection
$S_{xx} = \frac{I_{xx}}{C} = \frac{bd^2}{6}$	$\Rightarrow$	Section Modulus (In^3)	$\Rightarrow$	Bending Moment

b = width

d = depth

c = location of Neutral Axis

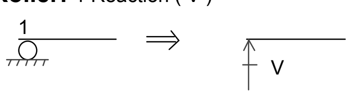


Radius of Gyration = $r = \sqrt{\frac{I}{A}}$

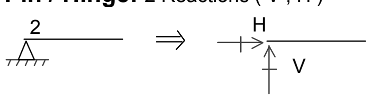
- If a Member is inadequate in Shear, increasing the Area (either Width (b) or Depth (d)) is effective.
- If a Member is inadequate in Deflection, increasing the Moment of Inertia (Width (b) is OK; but Depth (d) is cubed and) is much more effective in reducing Deflection.
- If a Member is inadequate in Bending, increasing the section modulus (width (b) is OK; but Depth (d) is squared and) is much more effective in reducing Bending.

Support Conditions

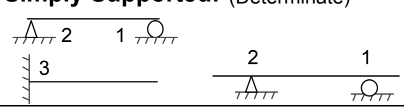
Roller: 1 Reaction (V)



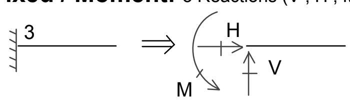
Pin / Hinge: 2 Reactions (V , H)



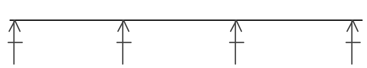
Simply Supported: (Determinate)



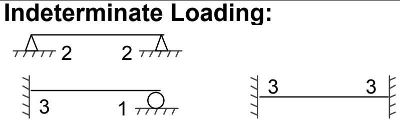
Fixed / Moment: 3 Reactions (V , H , M)



Continuous: Multiple Reactions



Indeterminate Loading:



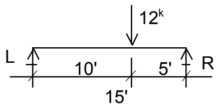
- Statically Determinate (Simply Supported) loading = three unknown reactions, and can be solved using the equation of Static equilibrium.
- Statically Indeterminate loading > 3 unknown Reactions Call your engineer.
- Pin/Hinged connections iclude most wood to wood, bolted steel, and precast concrete connections.
- fixed connections include most welded steel / steel connections and cast-in-place concrete.

CONCEPT	FORMULAE AND DIAGRAMS	COMMENTS

Shear and Bending Moment Diagrams

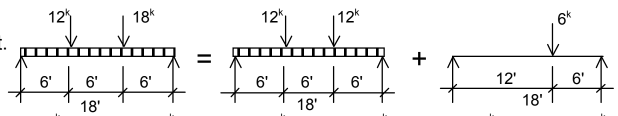
Example 1:

- L < R
- $L = \frac{5'}{15'} \times 12^k = 4^k$
- $R = \frac{10'}{15'} \times 12k = 8^k$



Example 2:

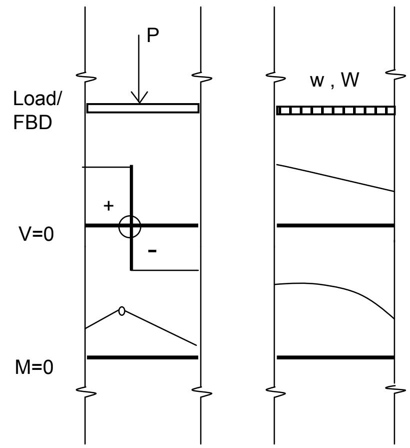
w = 1k/ft.
$W = 18^k$



$L = 23^k$ $R = 25^k$ | $L = 21^k$ $R = 21^k$ | $L = 2^k$ $R = 4^k$

$L = 21^k + 2^k$ $R = 21^k + 4^k$
$L = 23^k$ $R = 25^k$

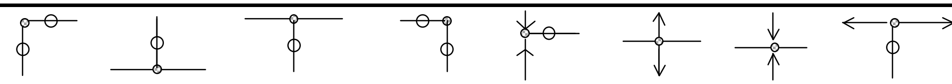
$L = 6'/18' \times 6^k = 2^k$
$R = 12'/18' \times 6^k = 4^k$

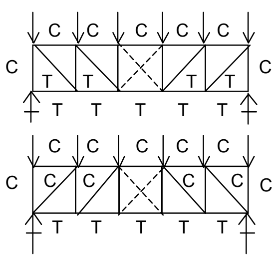


- M = Moment
- V =Shear
- Equilibruim = $\sum F_x = 0$; $\sum F_v = 0$; $\sum M_{Anv} = 0$
- Sum of Areas in Shear Diagram = Moment
- Magnitude of drop = Concentrated Load
- Between concentrated loads, Moment Diagram Slopes
- Uniform loads create gradual drop in Shear (straight line)
- Uniform loads create curve (downward cup) in Moment Diagram
- Overhangs and cantilevers will always have a negative Moment in Moment Diagram. Simply supported beams always have positive Moments
- V_{MAX} always occurs at support ⟹ Moment is minimum
- M_{MAX} occurs where V = 0
- Uniform load coefficient, w, = slope in Shear Diagram
- Point of Inflection (P.O.I.) is a point on the Moment Diagram where M = 0
- Point of Inflection only happens when a beam has an overhang
- If Loading Diagram (FBD) is symmetrical, then the Shear Diagram and the Moment Diagram are also symmetrical.
- Maximum Shear dictates how much Beam area is needed
- Maximum Moment dictates how much Bema Depth is needed
- If a hole must be punched out of a Beam to allow for passage of pipe or similar reduction, this must happen at a location of low Shear and low Bending Moment

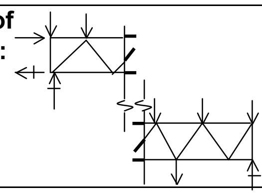
Trusses

Possible Zero Members

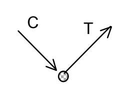


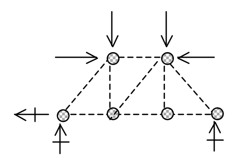


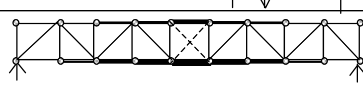
Method of Sections:



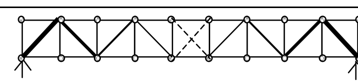
Method of Joints:







Top and Bottom Chord Stress
Stress increases towards middle



Web Stresses
Stress increases towards end panels

- A Truss is inherently stable due to triangulation
- Truss is stable in its own plane but needs bridging or cross-bracing perpendicular to its own plane
- All joints in an honest Truss are Pinned Joints
- Rigid Joints in a Truss will result in less Deflection than Pinned Joints (Advantage)
- Rigid Joints in a Truss will result in larger size members than Pinned Joint Trusses since members will have to resist V and M in addition to C or T (Disadvantage)
- Members carrying Tension can be much smaller than members carrying Compression
- m + 3 = 2 j ; where m = Number of Members
  j = Number of Joints
- Method of Joints is used to analyze Force / Stress in every member of a Truss
- Method of Joints is also used to analyze Force / Stress in a member that is close to a support (not in middle of truss)
- Method of Sections is used to analyze only a few (3 max) members of a truss
- After cutting a truss in 2 segments, each segment is in Equilibrium $\Sigma F_X = 0$; $\Sigma F_Y = 0$; $\Sigma M_{ANY} = 0$
- Concentrated Loads in a Truss must be applied at panel points; otherwise we have combined stresses (T or C + V and M)
- Joints that have three or less members framing into them, may potentially have Zero Members

CONCEPT	FORMULAE AND DIAGRAMS	COMMENTS

General Beam Design

MATERIAL: F_v, F_b, E; F_C, F_T, F_P

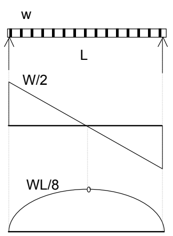
LOAD: L, w, W, P, FBD; V_{MAX}, M_{MAX}

GEOMETRY: A = bd ⟹ Shear; $I = bd^3/12$ ⟹ Deflection; $S = (bd^2)/6$ ⟹ Bending

DESIGN FOR SHEAR: $f_v < F_v$; $F_v \, \alpha \, \frac{V_{MAX}}{A_{MIN}}$

DESIGN FOR BENDING: $f_b < F_b$; $F_b = \frac{M_{MAX}}{S_{MIN}}$

DEFLECTION: $\Delta_{actual} = \frac{CONST \times (W \text{ or } P)(L \times 12''/ft.)^3}{EI}$; $\Delta_{actual} < \Delta_{allow}$

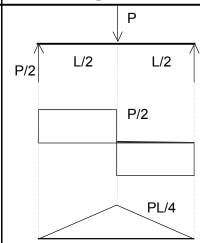


W = wL

$V_{MAX} = W/2$

$M_{MAX} = WL/8 = wL^2/8$

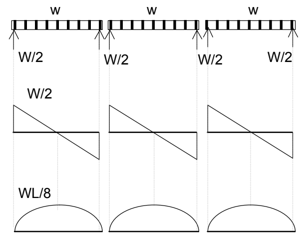
$\Delta_{MAX} = \frac{5}{384}\frac{WL^3}{EI} = \frac{5}{384}\frac{wL^4}{EI}$



$V_{MAX} = P/2$

$M_{MAX} = PL/4$

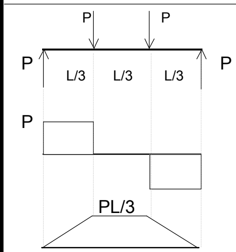
$\Delta_{MAX} = \frac{1}{48}\frac{PL^3}{EI}$



W = wL

$V_{MAX} = W/2$

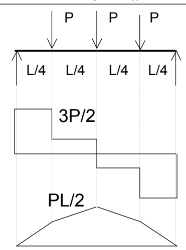
$M_{MAX} = WL/8 = wL^2/8$



$V_{MAX} = P$

$M_{MAX} = PL/3$

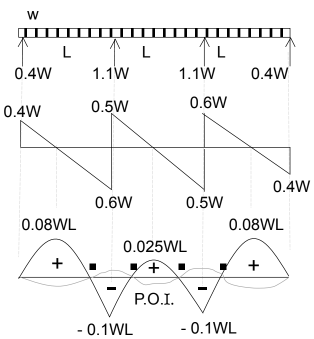
$\Delta_{MAX} = \frac{23}{648}\frac{PL^3}{EI}$



$V_{MAX} = 3P/2$

$M_{MAX} = PL/2$

$\Delta_{MAX} = \frac{19}{348}\frac{PL^3}{EI}$



W = w L

$V_{MAX} = 0.6\ W$

$M_{MAX}(+) = 0.08\ W\ L$

$M_{MAX}(-) = -\ 0.1\ W\ L$

Comments:

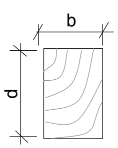
- Beam design must satisfy Shear, Bending Moment and Deflection requirements
- The Allowable Stress (F) of a species of wood or a Grade of steel depends on the material itself and is tabulated in Manuals and Building Codes
- The Actual Stress (f) is an outcome of the application of a load (W , P) on a member
- When a Load is applied perpendicular to the axis of a member (Normal Loading), Shear and Bending stresses develop
- The Strain associated with Bending is called Deflection and the deflected shape of a Beam is the inverse (upside/down) of the Moment Diagram
- When a load is applied along the axis of a member, Axial Compression and Tension Stresses develop
- The strain associated with Tension is Elongation and the strain associated with Compression is Shortening
- For the same Magnitude and span, a Uniform Load will cause less Deflection than a Concentrated Load for the same material and geometry
- The the same Load and Span, a Cantilever will deflect more than a simply supported beam
- For the same Load, Material and Geometry a slight increase in Span will create a huge increase in Deflection
- For the same Load and Span, an increase in the Modulus of Elasticity, E, (a stronger material), will result in less Deflection
- For the same Load and Span, an increase in the Moment of Inertia, I , (a deeper member) will result in less deflection
- The Points of Inflection on the Moment Diagram of the Continuous beam (Left) indicate the locations of curve reversal, and are the locations where reinforcing steel would be flipped from bottom to top of the beam.

Beams

WOOD BEAMS:

Shear: $F_V = \frac{3\ V_{MAX}}{2\ A_{MIN}}$

Bending: $F_b = \frac{M_{MAX}}{S_{MIN}}$

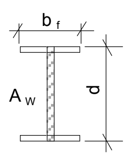


A = b x d

STEEL BEAMS:

Shear: $F_V = \frac{V_{MAX}}{A_{WEB}}$

Bending: $F_b = \frac{M_{MAX}}{S_{MIN}}$



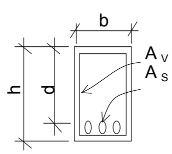
F_b= 24 KSI (full lateral support) ⟹ S_{xx} tables

F_b< 24 KSI (partial lateral support) ⟹ L_{UNB} , M-Charts

CONCRETE BEAMS:

Shear: Concrete: f'_c
b, d, f'_c
Stirrups: f_y
f_v, ϕ, A_v, spacing

Bending: Concrete: f'_c
b, d, f'_c
Rebars: f_y
f_y, (ϕ, # rebars), A_s



Comments:

- For all beams; $\Delta_{actual} = \frac{CONST.(W \text{ or } P)(L \times 12''/ft.)^3}{EI}$
- Allowable Deflecion is specified by model codes as a fraction of the span Δ_{allow} = L / 240, L / 360,...

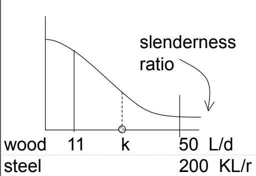
Columns

WOOD COLUMNS:

Slenderness:

$L_{UNB.}/d_{Least}$

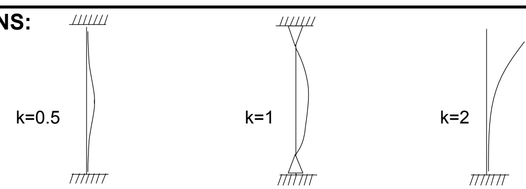
$k_{wood} = 0.671\sqrt{\frac{E}{F_c}}$



STEEL COLUMNS:

Slenderness:

$\frac{kL_{UNB.}}{r}$



Comments:

- F_C = P/A
- Long and thin (slender) columns tend to be governed by buckling
- Short and fat (chunky) columns tend to be governed by crushing